Year 2. Problem Set 34.

1. For his birthday party, Tim invited 8 boys. Every guest received a goody bag. Tim had three types of goody gifts: toy cars, chocolate bars and bouncing balls. While preparing the goody bags, Tim was constantly distracted by arriving guests. He added toy cars to some of the bags, but maybe not to every bag. The same happened to chocolate bars and bouncing balls. Could it be possible that each guest received a different goody gift?

2. x, y and z are integer numbers such that $x^2 + y^2 = z^2$. Prove that at least one of the numbers x, y and z is divisible by 3.

3. Find all such integer numbers that increase 5 times if their leftmost digit is relocated to the rightmost position.

4. Point A doesn't belong to a straight line a ($A \notin a$). Prove that it isn't possible to draw more than one perpendicular to the line a that contains point A. In other words, prove that if b and c are two lines perpendicular to a and containing A, then these two lines coincide.

5. Two parallel lines are drawn on a plane. N different points are marked on the first line, and M different points are marked on the second line.
    a. How many different straight lines containing 2 or more marked points can you draw?
    b. How many different rays starting at one marked point and containing another marked point can you draw?

6. What is the minimal number of kings that you could place on a chessboard in such a way that any three L-shaped squares contain at least one king? (Do not forget that you should not only show how to place the pieces, but also to prove that this is a minimal possible number).